FINITENESS OF TOPOLOGICAL ENTROPY FOR LOCALLY COMPACT ABELIAN GROUPS

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Abstract. The aim of the talk is to expose recent joint results with Anna Giordano Bruno and Francesco G. Russo on the finiteness of the topological entropy h_{top} for continuous endomorphisms of locally compact groups (here the topological entropy h_{top} is intended in the sense of Rufus Bowen [2]).

More precisely, in [1, 3] was initiated the study the class $\mathfrak{E}_{<\infty}$ of topological groups G such that all continuous endomorphisms of G have finite topological entropy, as well as its subclass $\mathfrak{E}_0$, of groups with vanishing topological entropy of their continuous endomorphisms. The paper [3] studied exclusively the compact case, which was facilitated by the fact that an Addition Theorem for continuous endomorphisms of compact groups was proved already by Sergey Yuzvinski [7] (in the metrizable case, see [1] for the general case). We say that *the Addition Theorem holds* for a triple G, ϕ, H of a topological group G, $\phi \in \mathrm{End}(G)$ and a ϕ-invariant closed normal subgroup H of G, if

$$h_{top}(\phi) = h_{top}(\phi\restriction_H) + h_{top}(\bar{\phi}_{G/H}), \tag{1}$$

where $\bar{\phi}_{G/H} \in \mathrm{End}(G/H)$ is induced by ϕ. This can be briefly resumed by saying that in the following commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & H & \xrightarrow{\iota} & G & \xrightarrow{\pi} & G/H & \longrightarrow & 0 \\
 & & \big\downarrow{\scriptstyle \phi\restriction_H} & & \big\downarrow{\scriptstyle \phi} & & \big\downarrow{\scriptstyle \bar{\phi}} & & \\
0 & \longrightarrow & H & \xrightarrow{\iota} & G & \xrightarrow{\pi} & G/H & \longrightarrow & 0
\end{array}$$

the topological entropy of the middle vertical arrow is the sum of the topological entropies of the remaining two vertical arrows.

The Addition Theorem is known to hold for totally disconnected locally compact groups and their topological automorphism of ([5]). However the validity of the Addition Theorem in the general case of locally compact groups and their continuous endomorphisms is not yet established even in the abelian setting, to the best of our knowledge. (A wrong proof of the Addition Theorem for locally compact abelian groups appeared in [6] – see [4] for more detail.)

We continue study the classes $\mathfrak{E}_{<\infty}$ and $\mathfrak{E}_0$ in the case of locally compact groups. In the abelian case we obtain a reduction of the problem to the case of periodic locally compact abelian groups, and then to locally compact abelian p-groups.

We show that locally compact abelian p-groups of finite rank belong to $\mathfrak{E}_{<\infty}$; moreover, those of them that belong to $\mathfrak{E}_0$ are precisely the groups with discrete maximal divisible subgroup. Furthermore, the topological entropy of the endomorphisms of the locally compact abelian p-groups of finite rank coincides with the logarithm of the scale.

The proofs of the main results use various versions of the Addition Theorem that were established *ad hoc*.

References

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